# Lecture notes on the electrodynamics of metals and superconductors

## Contents

# 1 Electrodynamics of metals: the Drude model

We begin by extending the Drude model at finite frequencies. This will allow us to account for the fundamental optical properties of metals. As we have seen in lecture 5, we assume the average scattering time between electrons and impurities to be $\tau$. In our previous analyses, we have grounded this parameter within a semiclassical and a fully quantum framework. However, for the sake of simplicity, we begin our discussion within a classical picture. As we have already emphasised, this approach captures the essence of transport phenomena, once the parameter $\tau$ is appropriately interpreted.

The equation of motion for the momentum $\boldsymbol{p}$ is

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -e\boldsymbol{E}(t) - \frac{\boldsymbol{p}}{\tau} \tag{1}$$

We will now discuss the dynamics induced by a time-dependent homogeneous electric field

$$\boldsymbol{E}(t) = \boldsymbol{E}(\omega)e^{-i\omega t} \tag{2}$$

looking for solutions for the momentum

$$\boldsymbol{p}(t) = \boldsymbol{p}(\omega)e^{-i\omega t} \tag{3}$$

Assuming this form of the solutions, we can compute the following:

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -i\omega\boldsymbol{p}(\omega)e^{-i\omega t} = -e\boldsymbol{E}(\omega)e^{-i\omega t} - \frac{\boldsymbol{p}(\omega)}{\tau}e^{-i\omega t} \tag{4}$$

$$-i\omega\boldsymbol{p}(\omega) = -e\boldsymbol{E}(\omega) - \frac{\boldsymbol{p}(\omega)}{\tau} \tag{5}$$

$$\boldsymbol{p}(\omega)\left(\frac{1}{\tau} - i\omega\right) = -e\boldsymbol{E}(\omega) \tag{6}$$

$$\boldsymbol{p}(\omega)\left(\frac{1 - i\omega\tau}{\tau}\right) = -e\boldsymbol{E}(\omega) \tag{7}$$

$$\boldsymbol{p}(\omega) = -\frac{e\tau}{1 - i\omega\tau}\boldsymbol{E}(\omega) \tag{8}$$

This calculation gives us access to a classical expression for the current density

$$\boldsymbol{j}(\omega) = -ne\boldsymbol{v}(\omega) = -ne\boldsymbol{p}(\omega)/m = \frac{ne^2\tau/m}{1 - i\omega\tau}\boldsymbol{E}(\omega) = \sigma(\omega)\boldsymbol{E}(\omega) \tag{9}$$

and, consequently, to the frequency-dependent Drude conductivity

$$\sigma(\omega) = \frac{ne^2\tau/m}{1 - i\omega\tau} = \frac{\sigma_{\mathrm{dc}}}{1 - i\omega\tau} \tag{10}$$

with

$$\sigma_{\mathrm{dc}} = \frac{ne^2\tau}{m} \tag{11}$$

### 1.1 Drude conductivity in linear response theory

Despite the simplicity of its derivation, the Drude conductivity captures the essential features of the frequency-dependent conductivity of metals. Moreover, it is a good starting point that can be naturally extended to obtain a more accurate description of many-body systems. We will now analyse its features. First of all, let's ensure that this form of the conductivity has the essential properties of a well-behaved response function. In light of linear response theory, we need to make sure that this response function is causal (causality of a time-dependent response function requires that there is no response before the stimulus arrives). In order to show that this is indeed the case for the Drude response, let's compute the time-dependent current response, as the Fourier transform of the frequency-dependent response:

$$\boldsymbol{j}(t) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \boldsymbol{j}(\omega) e^{-i\omega t} = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma(\omega) \boldsymbol{E}(\omega) e^{-i\omega t} \tag{12}$$

Next, we plug in the Fourier transform of the electric field

$$\boldsymbol{E}(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}t \boldsymbol{E}(t) e^{i\omega t} \tag{13}$$

to obtain

$$\boldsymbol{j}(t) = \int_{-\infty}^{+\infty} \mathrm{d}t' \underbrace{\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma(\omega) e^{-i\omega(t-t')}}_{\sigma(t-t')} \boldsymbol{E}(t') \tag{14}$$

$$\boldsymbol{j}(t) = \int_{-\infty}^{+\infty} \mathrm{d}t' \sigma(t-t') \boldsymbol{E}(t') \tag{15}$$

From this equation it is clear that the time-dependent current response is fully determined by the convolution of the time-dependent electric field $\boldsymbol{E}(t')$ with the response function

$$\sigma(t-t') = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma(\omega) e^{-i\omega(t-t')} \tag{16}$$

We will now compute this response function associated with the Drude form

$$\sigma^{\mathrm{Drude}}(t-t') = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\sigma_{\mathrm{dc}}}{1-i\omega\tau} e^{-i\omega(t-t')} = \tag{17}$$

$$= \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma_{\mathrm{dc}} \frac{i}{\omega\tau + i} e^{-i\omega(t-t')} = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\sigma_{\mathrm{dc}}}{\tau} \frac{i}{\omega + i/\tau} e^{-i\omega(t-t')} \tag{18}$$

For $t - t' > 0$, we can extend the integration to the lower half of the complex plane, where the integrand exhibits a simple pole at $\omega = -i/\tau$ (see figure 1a). Therefore, according to

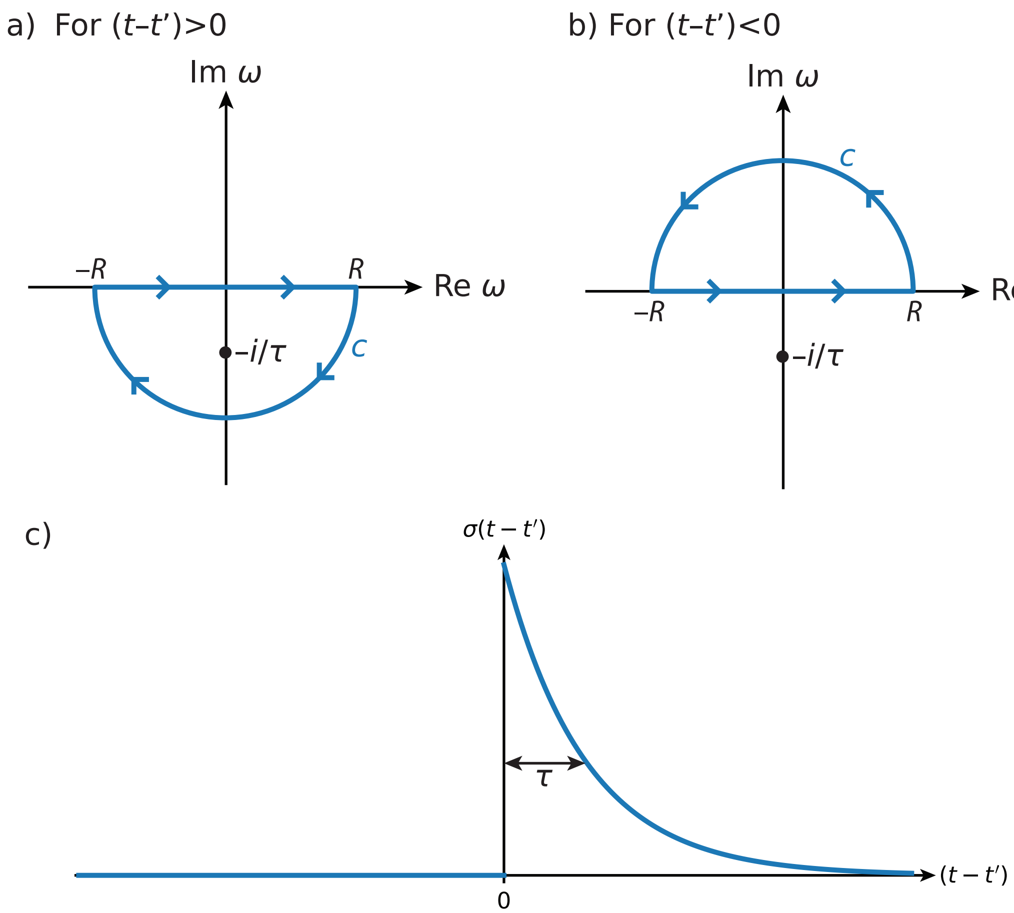


Figure 1:
Causality of the Drude response function

the Cauchy residue, we find

$$\sigma^{\mathrm{Drude}}(t-t') = \frac{\sigma_{\mathrm{dc}}}{\tau} e^{-(t-t')/\tau} \text{ for } t-t'>0 \tag{19}$$

For $t-t'<0$, we can extend the integration to the upper half of the complex plane, where the integrand exhibits no poles (see figure 1b). The Cauchy residue theorem yields

$$\sigma^{\mathrm{Drude}}(t-t') = 0 \text{ for } t-t'>0 \tag{20}$$

Therefore, we can write a simple expression for the response function, valid for all times

$$\sigma^{\mathrm{Drude}}(t-t') = \frac{\sigma_{\mathrm{dc}}}{\tau} e^{-(t-t')/\tau}\theta(t-t') \tag{21}$$

where $\theta(t-t')$ is the familiar Heaviside function. As a bonus result, this calculation also allows us to identify the Fourier transform of the Heaviside function (used in lecture 7) from

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\pm i}{\omega \pm i0^{+}} e^{-i\omega t} = \theta(\pm t) \tag{22}$$

Inspecting figure 1c, we can immediately appreciate that this response function ensures causality of the linear response. We will use it as a first approximation description of the electrodynamics of metals characterised by an elastic scattering time $\tau$.

## 1.2 Optical conductivity

The frequency-dependent Drude conductivity

$$\sigma(\omega) = \frac{ne^2\tau/m}{1-i\omega\tau} = \frac{\sigma_{\mathrm{dc}}}{1-i\omega\tau} \tag{23}$$

is characterised by a real and an imaginary part

$$\sigma_1(\omega) + i\sigma_2(\omega) = \frac{ne^2\tau}{m}\frac{1+i\omega\tau}{1+\omega^2\tau^2} \tag{24}$$

In the previous lecture we have emphasised the importance, in the dynamics of charged systems with long-range interactions, of collective modes manifesting at finite frequency called plasmons. We remind ourselves of their characteristic plasma frequency (expressed here in SI units, at variance with lecture 9 where Gaussian units were used):

$$\omega_p^2 = \frac{ne^2}{m\epsilon_0} \tag{25}$$

We can then write the real and imaginary part of the optical conductivity as

$$\mathrm{Re}[\sigma(\omega)] = \sigma_1(\omega) = \epsilon_0\omega_p^2\tau\frac{1}{1+\omega^2\tau^2} \tag{26}$$

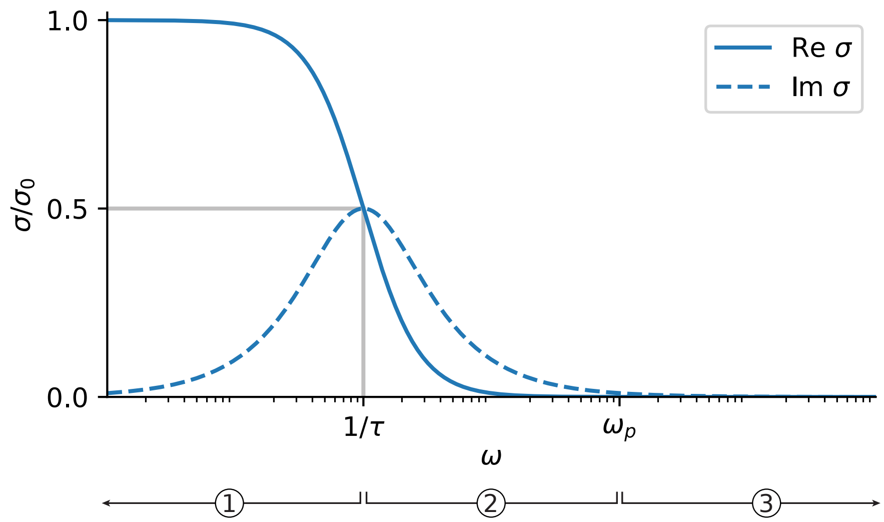


Figure 2: Optical conductivity of the Drude model

$$\mathrm{Im}[\sigma(\omega)] = \sigma_2(\omega) = \epsilon_0 \omega_p^2 \tau \frac{\omega\tau}{1+\omega^2\tau^2} \tag{27}$$

Evidently, 2 parameters characterise the optical conductivity: the elastic scattering time $\tau$ and the plasma frequency $\omega_p$. Realistic experimental values for these parameters in metals are elastic scattering times in the order of $10^{-15}$ s up to $10^{-12}$ s (depending on the amount of disorder present), and plasma frequencies from the visible up to the UV range, with associated photon energies in the eV range. Therefore, we can assume the inequality

$$\frac{1}{\tau} < \omega_p \tag{28}$$

to hold. This allows us to identify 3 characteristic regimes of the optical conductivity, manifesting within different frequency windows (see figure 2): (1) the quasi dc regime, (2) the relaxation regime and (3) the transport regime

#### 1.2.1 Quasi dc regime

$$\boxed{\omega < \frac{1}{\tau}}$$

Here the system is being probed at frequencies below the characteristic scattering rate. We find that the real part of the conductivity has a negligible frequency dependence and approaches the dc value

$$\sigma_1 \simeq \sigma_{\mathrm{dc}} \tag{29}$$

The imaginary part of the conductivity depends linearly on frequency

$$\sigma_2 \simeq \sigma_{\mathrm{dc}}\omega\tau \tag{30}$$

This regime is characteristic of the microwave and THz response of disordered metals

### 1.2.2 Relaxation regime

$$\boxed{\frac{1}{\tau} < \omega < \omega_p}$$

In this regime, both the real and imaginary part of the conductivity are characterised by a strong frequency dependence

$$\sigma_1 \simeq \frac{\sigma_{\mathrm{dc}}}{\omega^2\tau^2} \tag{31}$$

$$\sigma_2 \simeq \frac{\sigma_{\mathrm{dc}}}{\omega\tau} \tag{32}$$

Throughout this regime, the imaginary part represents the dominant response.

### 1.2.3 Transport regime

$$\boxed{\omega > \omega_p}$$

In this regime, the system is probed at frequencies that are much faster than the characteristic response times. Therefore, the electron system cannot respond fast enough to these rapid electric field variations and, effectively, the medium becomes (partly) transparent to the propagation of electromagnetic waves. This fact is underscored in figure 3, where the frequency dependence of the reflectivity is shown. In this regime, the real and imaginary parts of the conductivity continue to show the same frequency dependence observed in the relaxation regime ($\sigma_1 \propto \omega^{-2}$ and $\sigma_2 \propto \omega^{-1}$).

## 1.3 Kramers-Kronig relations

The conductivity being analysed here is part of a broader family of frequency-dependent susceptibilities $G(\omega)$. In general these are complex quantities with a real part, $\mathrm{Re}G(\omega)$, representing the attenuation of a stimulus and an imaginary part, $\mathrm{Im}G(\omega)$, the phase difference between the excitation provided and the response recorded. In order to represent the response of a system that conforms to the principle of causality ($G(t-t') = 0$ for $t-t' < 0$), the frequency-dependent susceptibility $G(\omega)$ must possess a mathematical structure that is encoded in the so-called Kramers-Kronig relations. In order to unveil this structure, we consider a linear response of a field $X$ to a pertubation $f$, described by the response function, or susceptibility, $G$:

$$X(\boldsymbol{r}, t) = \int\int_{-\infty}^{+\infty} G(\boldsymbol{r}, \boldsymbol{r}', t, t') f(\boldsymbol{r}', t') \mathrm{d}\boldsymbol{r}' \mathrm{d}t' \tag{33}$$

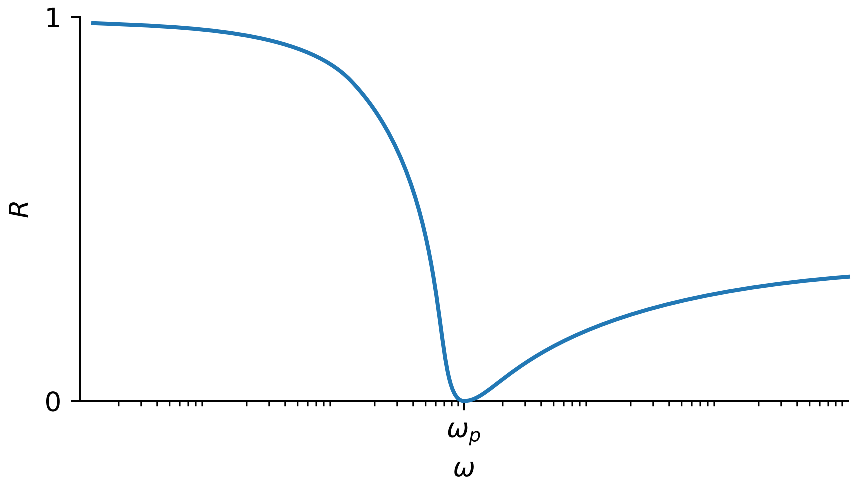


Figure 3: Frequency-dependent reflectivity in the Drude model

We will assume a local approximation for the susceptibility

$$G(\boldsymbol{r}, \boldsymbol{r}', t, t') = \delta(\boldsymbol{r} - \boldsymbol{r}')G(t - t') \tag{34}$$

and we request that our theory respects causality (no response before the stimulus arrives):

$$G(t - t') = 0 \text{ for } t - t' < 0 \tag{35}$$

The Fourier responses are

$$f(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}t f(t) e^{i\omega t} \tag{36}$$

$$G(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}t G(t - t') e^{i\omega(t-t')} \tag{37}$$

Therefore, we find the usual convolution theorem

$$X(\omega) = \int \mathrm{d}t e^{i\omega t} \left[G(t - t') f(t') \mathrm{d}t'\right] \tag{38}$$

$$= \int \mathrm{d}t' f(t') e^{i\omega t'} \left[G(t - t') e^{i\omega(t-t')} \mathrm{d}t\right] \tag{39}$$

$$= G(\omega) f(\omega) \tag{40}$$

We now consider $\omega \in \mathbb{C}$

$$\omega = \omega_1 + i\omega_2 \tag{41}$$

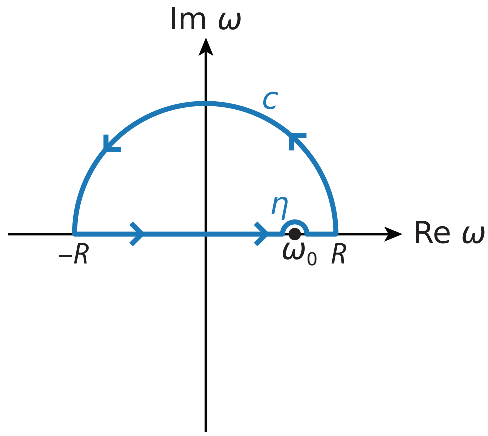


Figure 4: Integration contour utilised to derive the Kramers-Kronig relations

Upon inspection, we can verify that, for $t - t' > 0$, $G(\omega)$, defined as

$$G(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}t G(t-t') e^{i\omega_1(t-t')} e^{-\omega_2(t-t')} \tag{42}$$

is bounded in the upper-half of the $\omega$ complex plane. Consequently, it is instructive to consider the integral of the function $G(\omega)/(\omega - \omega_0)$ over a closed contour within this plane, as illustrated in figure 4. According to Cauchy's theorem, we have

$$\oint \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' = 0 \tag{43}$$

Therefore, we find

$$\oint \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' = \lim_{R\to\infty} \int_{-R}^{+R} \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' + \lim_{R\to\infty} \int_c \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' + \lim_{\eta\to 0} \int_\eta \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' = 0 \tag{44}$$

We note that, thanks to the fact that $G(\omega)$ is bounded in the region of interest of the complex plane, we have

$$\lim_{R\to\infty} \int_c \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' = 0 \tag{45}$$

Moreover, Cauchy's residue theorem allows us to compute

$$\lim_{\eta\to 0} \int_\eta \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' = -\frac{1}{2} 2\pi i \lim_{\omega\to\omega_0} (\omega - \omega_0) \frac{G(\omega)}{\omega - \omega_0} = -\pi i G(\omega_0) \tag{46}$$

The sign of the residue is due to the clockwise integration and the factor 1/2 is due to the semicircle contour. Plugging back in these two results, we find

$$\oint \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' = \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G(\omega')}{\omega' - \omega_0} \mathrm{d}\omega' - \pi i G(\omega_0) = 0 \tag{47}$$

We have come thus to the useful relation

$$G(\omega) = \frac{1}{\pi i} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G(\omega')}{\omega' - \omega} \mathrm{d}\omega' \tag{48}$$

that can be used to derive a structure that links the real and imaginary part of a response function

$$G(\omega) = G_1(\omega) + iG_2(\omega) \tag{49}$$

that conforms to causality. Indeed, we have found

$$G_1(\omega) + iG_2(\omega) = \frac{1}{\pi i} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G_1(\omega')}{\omega' - \omega} \mathrm{d}\omega' + \frac{1}{\pi i} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{iG_2(\omega')}{\omega' - \omega} \mathrm{d}\omega' \tag{50}$$

$$= \frac{1}{\pi} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G_2(\omega')}{\omega' - \omega} \mathrm{d}\omega' - \frac{i}{\pi} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G_1(\omega')}{\omega' - \omega} \mathrm{d}\omega' \tag{51}$$

Therefore, we can separately identify the real and imaginary parts of the response function as

$$G_1(\omega) = \frac{1}{\pi} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G_2(\omega')}{\omega' - \omega} \mathrm{d}\omega' \tag{52}$$

$$G_2(\omega) = -\frac{1}{\pi} \mathrm{pv} \int_{-\infty}^{+\infty} \frac{G_1(\omega')}{\omega' - \omega} \mathrm{d}\omega' \tag{53}$$

These are known as the Kramers-Kronig relations and they formalise the idea that causality ($G(t - t') = 0$ for $t - t' < 0$) requires that an attenuation ($G_1$), manifesting at frequency $\omega$, must be accompanied by a distribution of phase shifts ($G_2$) occurring at frequencies $\omega'$ (and vice versa).

## 1.4 Extended Drude model

As we have discussed previously, the effects of interactions can be described by the spectral function

$$A(k, \omega) = -\frac{1}{\pi} \frac{\mathrm{Im}\Sigma(k, \omega)}{(\omega - \xi(k) - \mathrm{Re}\Sigma(k, \omega))^2 + (\mathrm{Im}\Sigma(k, \omega))^2} \tag{54}$$

that introduces a frequency dependence of the scattering time

$$1/\tau(\omega) = -\mathrm{Im}\Sigma(\omega) \tag{55}$$

and a renormalisation of the effective mass.

$$m^* = \left(1 - \frac{\partial \mathrm{Re}\Sigma}{\partial \omega}\bigg|_{\omega=\epsilon(k)}\right) m \tag{56}$$

These effects can be introduced in the Drude conductivity

$$\sigma(\omega) = \frac{ne^2/m}{1/\tau - i\omega} \tag{57}$$

through the extended Drude model

$$\sigma(\omega) = \frac{\epsilon_0 \omega_p^2}{1/\tau(\omega) - i\omega m^*(\omega)/m} \tag{58}$$

This model contains the changes introduced to scattering by many-body interactions, by means of a frequency-dependent resistive response. Moreover, it contains the changes introduced to the effective mass, as a renormalisation of the Fermi velocity by $m^*/m$ that changes the inductive response.

## 2 Electrodynamics of superconductors: the London equations

The main experimental observations pertaining to superconductors are the following:

1. Vanishing of the electrical resistivity below a critical temperature $T_c$ (e.g. 300 mK in electron-doped $SrTiO_3$, 1.2 K in Al, 9.2 K in Nb, 39 K in $MgB_2$ or 90 K in hole-doped $YBa_2Cu_3O_7$).

2. Perfect diamagnetism (Meissner effect): below $T_c$, magnetic fields below a critical field $B_c$ do not penetrate a superconductor. The magnetic susceptibility in this regime, in an idealised case, is -1 (SI units).

3. The specific heat shows, below $T_c$, an activated behaviour of the type

$$C(T) \propto e^{-\Delta/kT} \tag{59}$$

4. The one-particle density of states shows, below $T_c$, a vanishing density of states (a gap), in an energy window, around the Fermi energy, of the order $\Delta$.

In this lecture, we will give a phenomenological account of the first two experimental observations by means of the London equations. This will allow us to provide a basic description of the electrodynamics of superconductors. In lecture 12 we will introduce a microscopic theory of superconductivity that will provide a rational basis for observations 3) and 4).

The starting point of the derivation of the phenomenological London equations is the assumption that we can describe a superconductor as the coexistence of two fluids: one fluid of normal state quasiparticles, with density $n_{qp}$, whose dynamics are described by the (extended) Dude model, and a second fluid of superconducting electrons, with density $n_s$, that are able to carry an electrical current without dissipation (supercurrent). The sum of these two contributions is equal to $n$, the normal state density above $T_\mathrm{c}$. However, the relative weight of the two components varies with temperature:

$$n_{qp}(T) + n_s(T) = n \tag{60}$$

The electric field-induced acceleration of the non-dissipative electrons is described by the equation

$$m\frac{\mathrm{d}\boldsymbol{v}_s}{\mathrm{d}t} = -e\boldsymbol{E} \tag{61}$$

and for the corresponding current density $\boldsymbol{j}_s = -n_s e\boldsymbol{v}_s$ we will have

$$\frac{\mathrm{d}\boldsymbol{j}_s}{\mathrm{d}t} = \frac{n_s e^2}{m}\boldsymbol{E} \tag{62}$$

Since we have for the total time derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}\nabla \tag{63}$$

with the first contribution being much larger than the second, we neglect the second contribution, arriving at the first London equation:

$$\boxed{\frac{\partial\boldsymbol{j}_s}{\partial t} = \frac{n_s e^2}{m}\boldsymbol{E}} \tag{64}$$

This expression is consistent with the Drude model in the limit of infinite scattering time:

$$\lim_{\tau\to\infty}\sigma_{\mathrm{Drude}}(\omega) = \frac{ne^2}{m}\pi\delta(\omega) + i\frac{ne^2}{m}\frac{1}{\omega} \tag{65}$$

The real part represents an infinite dc conductivity (vanishing resistivity) through a delta distribution, while the imaginary contribution represents the kinetic inductance of a conductor without scattering. Therefore, the first London equation essentially reflects the absence of scattering. We plug this result into Faraday's law of induction

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \tag{66}$$

to obtain

$$\frac{\partial}{\partial t}\left(\nabla\times\frac{m}{n_s e^2}\boldsymbol{j_s}+\boldsymbol{B}\right)=0 \tag{67}$$

In principle this equation admits trivial solutions with a static magnetic field and a static current density. However, we know from the experimental observation 2) that, at the interior of a superconductor, a magnetic field is completely expelled. Therefore, on these phenomenological grounds, we take the solution

$$\boxed{\nabla\times\boldsymbol{j_s}=-\frac{n_s e^2}{m}\boldsymbol{B}} \tag{68}$$

This is the second London equation, that describes a conductor without dissipation and with perfect diamagnetism, a superconductor. Using this equation and Ampère's law (neglecting the displacement current)

$$\nabla\times\boldsymbol{B}=\mu_0\boldsymbol{j}_s \tag{69}$$

we can capture the essential features of the Meissner effect. Indeed, we find

$$\nabla\times\nabla\times\boldsymbol{B}=-\frac{\mu_0 n_s e^2}{m}\boldsymbol{B} \tag{70}$$

By remembering that

$$\nabla\times\nabla\times\boldsymbol{B}=\nabla\nabla\cdot\boldsymbol{B}-\nabla^2\boldsymbol{B}=-\nabla^2\boldsymbol{B} \tag{71}$$

we find

$$\nabla^2\boldsymbol{B}=\frac{\mu_0 n_s e^2}{m}\boldsymbol{B}=\frac{1}{\lambda^2}\boldsymbol{B} \tag{72}$$

and similarly

$$\nabla^2\boldsymbol{j}_s=\frac{\mu_0 n_s e^2}{m}\boldsymbol{j}_s=\frac{1}{\lambda^2}\boldsymbol{j}_s \tag{73}$$

These equations indicate that the magnetic field and current density penetrate in a superconductor only within a layer of thickness $\lambda$, defined as

$$\lambda(T)=\sqrt{\frac{m}{\mu_0 n_s(T)e^2}} \tag{74}$$

$\lambda$ is known as the penetration depth. The zero temperature limit

$$\lambda(0)=\sqrt{\frac{m}{\mu_0 n e^2}} \tag{75}$$

represents the characteristic penetration length of a magnetic field and of a supercurrent into a superconductor. As the fraction of superconducting electrons decreases, and eventually vanishes when we approach $T_{\rm c}$, the penetration depth shows a divergent behaviour.

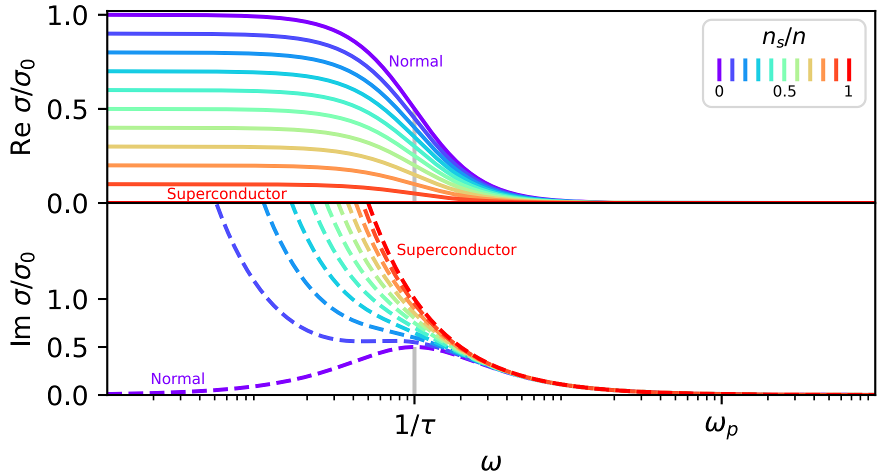


Figure 5: Two-fluid model for the conductivity of a superconductor

## 2.1 Optical conductivity

Starting form the two-fluid model, we can describe the conductivity of a superconductor as the sum of a quasiparticle contribution and a superconducting contribution:

$$\sigma(\omega) = \frac{\epsilon_0 \omega_p^2}{1/\tau(T) - i\omega} \frac{n_{qp}(T)}{n} + \frac{1}{\mu_0 \lambda^2(0)} \left[ \pi \delta(\omega) + \frac{i}{\omega} \right] \frac{n_s(T)}{n} \tag{76}$$

For the quasiparticle contribution we have taken the Drude model with a temperature-dependent scattering time (a minimal extended model) and for the superconducting contribution the Drude response in the limit of infinite scattering time. Figure 5 shows the prediction of the two-fluid model for the real and imaginary part of the conductivity calculated for different values of the superconducting fluid fraction. Here the effect of superconductivity is a prominent increase of the inductive response at frequencies below $1/\tau$, with a pronounced $\propto 1/\omega$ dependence.

While this model is a reasonable starting point for the description of the electrodynamics of superconductors, it fails to account for elementary absorption processes associated with a gapped single-particle density of states. These effects produce a significant correction to the $\sigma_1$ response and they are described by a more advanced theory elaborated by Mattis and Bardeen (Phys. Rev. 111, 412 (1958)).